4 The new Keynesian model

4.1 Foundations: a classical monetary model (household block)

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The New Keynesian model

- Classical monetary models - monetary policy has no role in determining real variables
- The fact that monetary policy has no role contrasts with evidence that monetary policy has real effects on the economy
- One reason for monetary policy to have real effect is the existence of nominal price rigidities - New keynesian model
- We will start though with a classical monetary model and then introduce frictions

A Simple Classical Monetary Model (Gali, 2008, Chapter 2)

The Model has the following features:

- The economy has two sectors: households and firms
- Households and firms are modelled in terms of representative agents
- Prices and wages are flexible
- The problem consists in maximizing the utility function of the representative household subject to its budget constraint and the behaviour of the firms

The objective function

The household tries to maximize expected utility:

$$E_0 \sum_{t=0}^{\infty} \beta^t U(C_t, N_t) \tag{1}$$

where C_t is consumption in each period and N_t is hours worked or employment. β is a discount factor linked to agent's time-preference.

Budget constraint

The household is subject to the following period budget constraint:

$$P_t C_t + Q_t B_t \leq B_{t-1} + W_t N_t - T_t \quad (2)$$

where:

- P_t - price of consumption good
- B_t - quantity of one-period riskless discount bond purchased in period t and maturing in t+1(value $=1$ at maturity).
- $\lim_{T\to\infty} E_t B_T \geq 0$ - no Ponzi games allowed
- Q_t - Price of bond
- W_t - nominal wage
- T_t - lump sum additions, subtractions (e.g. dividends, lump-sum taxes)

Optimization

The household is assumed to take P_t, W_t and B_t as given and decides how much to consume and work (C_t, N_t). Galí derives optimality conditions based on a variational argument. If the agent is maximizing then it must be the case that:

$$U_{c,t} dC_t + U_{n,t} dN_t = 0 \tag{3}$$

for pairs (dC_t, dN_t) satisfying the budget constraint:

$$P_t dC_t = W_t dN_t \tag{4}$$

Otherwise it would be possible to improve utility.

Intra-temporal optimizing condition

Combining the equations:

$$U_{c,t}dC_t + U_{n,t}dN_t = 0 \tag{5}$$

$$P_t dC_t = W_t dN_t \tag{6}$$

One obtains the intra-temporal condition:

$$\boxed{-\frac{U_{n,t}}{U_{c,t}} = \frac{W_t}{P_t}} \tag{7}$$

Optimization of households problem

Besides an intra-temporal dimension there is an inter-temporal problem to solve. In fact, the household will be optimising when the utility loss of forfeiting consumption today equals the discounted expected utility gain of consumption tomorrow:

$$U_{c,t} dC_t = -\beta E_t \left[U_{c,t+1} dC_{t+1} \right] \quad (8)$$

Note that such shifting of consumption into the future is made possible through saving (using the bonds). This must respect the following restriction:

$$P_{t+1} dC_{t+1} = -\frac{P_t}{Q_t} dC_t \quad (9)$$

Intertemporal optimizing condition

Solving both equations:

$$U_{c,t} dC_t = -\beta E_t \left[U_{c,t+1} dC_{t+1} \right] \tag{10}$$

$$P_{t+1} dC_{t+1} = -\frac{P_t}{Q_t} dC_t \tag{11}$$

One obtains the inter-temporal condition:

$$\boxed{Q_t = \beta E_t \left[\frac{U_{c,t+1}}{U_{c,t}} \frac{P_t}{P_{t+1}} \right]} \tag{12}$$

Optimization of households problem

Let us assume the following utility function:

$$U(C_t, N_t) = \frac{C_t^{(1-\sigma)}}{1-\sigma} - \frac{N_t^{1+\varphi}}{1+\varphi} \tag{13}$$

Using the above optimality conditions:

$$\frac{W_t}{P_t} = C_t^{\sigma} N_t^{\varphi} \tag{14}$$

$$Q_t = \beta E_t \left[\left(\frac{C_{t+1}}{C_t}\right)^{-\sigma} \frac{P_t}{P_{t+1}} \right] \tag{15}$$

Log linearization

Non-linearities in general equilibrium models create difficulties in solving the models. As a result it is usual to work with log linearizations around the steady state, whenever justified. The log linearizations are usually obtained from a first order Taylor expansion around the steady state. Recall that for a univariate function f the first order approximation of f(X) around a value a would be:

$$f(X) \approx f(a) + f'(a)(X - a)$$

Log-linearization

Suppose you have a function of the following form:

$$(X_t, Y_t) = g(Z_t) \tag{16}$$

This equation must also hold in steady state:

$$f(X, Y) = g(Z) \tag{17}$$

Log linearizations

We can rewrite the previous equation as:

$$log(f(e^{log(X_t)}, e^{log(Y_t)}) = log(g(e^{log(Z_t)})) \tag{18}$$

Taking a first order Taylor expansion around the steady state, and denoting $\hat{x}_t = log(X_t) - log(X)$. The left hand side becomes:

$$log(f(X, Y)) + \frac{1}{f(X, Y)}[f_x(X, Y)X\hat{x}_t + f_y(X, Y)Y\hat{y}_t] \tag{19}$$

The right hand side is:

$$log(g(Z)) + \frac{1}{g(Z)}[g_z(Z)Z\hat{z}_t] \tag{20}$$

So the Taylor expansion approximation is:

$$\boxed{f_x(X, Y)X\hat{x} + f_y(X, Y)Y\hat{y} = g_z(Z)Z\hat{z}} \quad (21)$$

Log linearized household block

Going back to our example the intratemporal condition can be log-linearised simply by taking logs (lower case letters are logged):

$$\boxed{w_t - p_t = \sigma c_t + \varphi n_t} \tag{22}$$

And the intertemporal log linearised version (assuming constant inflation and consumption growth in steady state):

$$\boxed{c_t \simeq E_t c_{t+1} - \frac{1}{\sigma}(i_t - E_t \pi_{t+1} - \rho)} \tag{23}$$

where

- $i_t = -logQ_t$ is the nominal interest rate making use of the result $Q_t = \frac{1}{(1+yield)}$
- $\rho = -log\beta$ is the representative household's discount rate
- $\pi_{t+1} = p_{t+1} - p_t$ is inflation